A GENERALIZED EQUATION FOR THE DISTRIBUTION OF CIRCUMFERENTIAL VELOCITIES IN VORTICES

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A generalized equation is obtained for the distribution of circumferential velocities in vortices over a horizontal orifice. The computed values of the circumferential velocities are compared with experimental results for vortices of various intensities.

Experimental investigations of the rotation of fluids in vortices [1], twisted jets [2], hydraulic cyclones [3], vortex chambers [4], and atmospheric vortices [5] have shown that the central region of the vortex rotates as a rigid body, while at the periphery the fluid rotates dynamically.

In the case of a flow which is symmetrical with respect to the vertical axis, passing through the center of the horizontal drainage orifice in the bottom, and when the flow conditions are established, the distribution of circumferential velocities in the radial direction is described by the equation [6]

$$\frac{v}{\sqrt{gH}} = f\left(\mu, \frac{r}{d}\right). \tag{1}$$

Here, when $r \geq 1.5d$, we have [7]

$$\frac{v}{\sqrt{gH}}\left(\frac{r}{d}\right)^k = \varphi(\mu), \tag{2}$$

where, for large Reynolds numbers $k \to 1$. For $r < 0.5d$ we have the approximate equation

$$\frac{v}{r} = \text{const},$$

or in nondimensional form

$$\frac{v}{\sqrt{gH}} \frac{d}{r} = \text{const}. \tag{3}$$

Equation (2), when $k = 1$, and (3) hold as $r \to \infty$ and $r \to 0$. Hence, for intermediate values of r it is reasonable to assume that

$$\frac{v}{\sqrt{gH}} = \frac{\psi(\mu)}{\frac{d}{r} + a\frac{r}{d}}. \tag{4}$$

Equation (4) reduces to (2) when $r \to \infty$, and to (3) when $r \to 0$.

To determine the coefficient a we used the experimental results of [6, 7]. Calculations for various arbitrary values of a showed that in the range $\mu = 0.15$–0.60 we have

$$\psi(\mu) = a(0.825 - 1.087\,\mu). \tag{5}$$

Using the method of least squares and (5) we were able to determine the coefficient a, which had the value $a = 4$. Thus, the equation for the circumferential velocities in a vortex has the form

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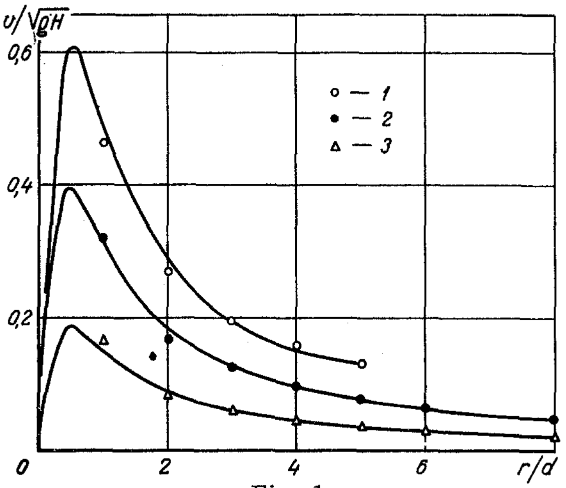

Fig. 1

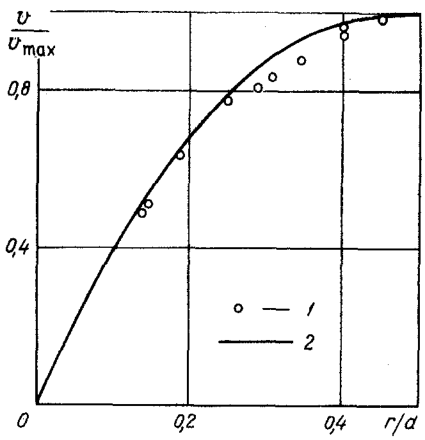

Fig. 2

Fig. 1. Comparison of computed and experimental values of the circumferential velocities in vortices [1) μ = 0.2; 2) 0.4; 3) 0.6].

Fig. 2. Distribution of the circumferential velocities near the axis of rotation [1) Anwar's data; 2) from (8)].

$$\frac{v}{\sqrt{gH}}=\frac{3.30-4.35\mu}{\frac{d}{r}+4\frac{r}{d}}. \tag{6}$$

We find the radius of rotation at which the circumferential velocity has its maximum value. To do this we determine the minimum value of the denominator on the right side of (6):

$$\frac{\partial}{\partial r}\left(\frac{d}{r}+4\frac{r}{d}\right)=-\frac{d}{r^2}+\frac{4}{d}=0,$$

from which r = d/2. Here

$$\frac{v_{\max}}{\sqrt{gH}}=0.825-1.087\mu\ , \tag{7}$$

and

$$\frac{v}{v_{\max}}=\frac{4}{\frac{d}{r}+4\frac{r}{d}}. \tag{8}$$

Thus, the maximum velocity for a given vortex, determined by (7), is at a distance of half the diameter of the orifice from the axis of rotation of the fluid. This conclusion is confirmed by the experimental investigations of Anwar [1]. Figure 1 gives a comparison of the values of $v/(gH)^{1/2}$ measured in our experiments [6, 7] and computed from (6) for several values of the flow-rate coefficient, while Fig. 2 compares the values of v/v_{max} measured by Anwar [1] and computed from (8) for $r \le d/2$. The graphs show satisfactory agreement between the experimental and computed values of the circumferential velocity.

NOTATION

- v is the circumferential velocity at distance r from axis of rotation;
- d is the diameter of drainage orifice;
- H is the pressure head;
- μ is the flow-rate coefficient at orifice;
- g is the acceleration due to gravity;
- a is the proportionality factor.

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